SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that the general solution of the linear PDE $Pp + Qq = R$ can be written in the form $F(u,v) = 0$, where F is an arbitrary function and $u(x,y,z) = c_1$ and $V(x,y,z) = c_2$ from a solution of $\frac{dx}{p(x,y,z)} = \frac{dy}{Q(x,y,z)} = \frac{dz}{R(x,y,z)}$.

17. Obtain the canonical from for elliptic equation.

18. Find the solution of Laplace equation in cylindrical co-ordinates.

19. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At the time $t = 0$ one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature, the other end remains thermally insulated. Find the temperature distribution $\theta(x,t)$.

20. Derive the vibration of the cylinder membrane.

APRIL/MAY 2023

GMA23/DMA23 — PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours

Maximum : 75 marks

Shanmuga Industries Arts & Science College
T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define orthogonal surface.
2. Write down Charpit's method.
3. What are the adjoint operators?
4. Write down the canonical forms.
5. Give the Poisson equation.
6. Write Neumann problems for rectangle.
7. Define delta function.
8. Write diffusion equation.
9. Give Duhamel's principle.
10. Write the two dimensional wave equation.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Form the partial differential equation by eliminating the constants from $z = ax + by + ab$.

Or

(b) Find the general integral of $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.

12. (a) Find the characteristics of the equation $u_{xx} + 2u_{xy} + \sin^2 x u_{yy} + u_y = 0$ when it is of hyperbolic type.

Or

(b) Derive the canonical form for hyperbolic equation.

13. (a) Show that the velocity potential for an irrotational flow of an incompressible fluid satisfies the Laplace solution.

Or

(b) Obtain the solution of Laplace equation in a cylindrical co-ordinates.

14. (a) In an one dimensional infinite solid $-\infty < x < \infty$, the surface $a < x < b$ is initially maintained at temperature T_0 and at zero temperature every where outside the surface. Prove that $T(x,t) = \frac{T_0}{2}\left[erf\left(\frac{b-x}{\sqrt{4at}}\right) - erf\left(\frac{a-x}{\sqrt{4at}}\right)\right]$ where *erf* is a error function.

Or

(b) Find the temperature $T(r,t)$ in the infinite cylinder $0 \le r \le a$ when the initial temperature is $T(r,\theta) = f(r)$ and surface $r = a$ is maintained at $\theta°$ temperature.

15. (a) Derive the D'Alembert's solution of the initial value problem.

Or

(b) Obtain the periodic solution of the wave equation in the form $u(x,t) = Ae^{i(kx \pm wt)}$, where $i = \sqrt{-1}$, $k = \pm\frac{w}{c}$, A is a constant.